Black Holes in Brief: A Creative Introduction

David A. Nichols

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Preface

This document takes a different tack towards communicating science to a non-technical audience. It does so by first presenting a new, creative writing by the author related to black holes. Specifically, some of the unconventional features of the writing—the unusual numbering of the stanzas, the placement of the text, the unfamiliar names, and the surprising themes—are subsequently elucidated in a pedagogical text following the creative writing. The level of detail provided in the explanatory text is intended to be sufficient for a non-expert to understand the main ideas and themes treated in the verse. It is not intended to be an exhaustive summary of black holes. There are references to technical articles and non-technical books where interested readers can pursue the subjects in greater detail.

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To Schwarzschild, from Finkelstein, Kruskal and Szekeres

II

This is beyond your universe.
You were fortunate
to never venture here.
Once inside, your old world
narrows and fades.
Nothing keeps space from closing in
with the inexorability of time's flow.
Distance becomes
the measure of time,
and time is short;
it is not long until
all that enters this universe
meets its end at a point
.

IV

This is *not* your universe,
though it looks much like it:
more silence, darkness, and emptiness,
and gravity pervading throughout.
Here too is an infinite domain
and a mysterious interior.
Inside there, you could unite briefly
with an ill-fated doppleganger
from another universe
in a location that feels
like the passage of time
measured with a distance
from a center
that can, and will, hold.

I

This *is* your universe:
silent, dark, and empty;
gravity in its purest form.
You could wander through it
for an eternity,
over its infinite space.
Its core was not known to you
and you did not venture there;
but not everything
can avoid it.
Though had you lived
to discover it before we had,
would you have adopted it
into your philosophy?

III
.
This too is beyond your universe,
and what you contemplated.
It starts at point.
You cannot reside here;
your expulsion is inevitable,
like the ticks of the clock.
No force can keep you still;
you must enter your universe,
or one unknown.

Introduction

The work on the previous page envisions a fictitious correspondence, in verse, from three of the four people whose names are mentioned in the title—Finkelstein, Kruskal and Szekeres—to the fourth, Schwarzschild. These four physicists all played an important role in discovering and understanding the nature of non-rotating black holes. Black holes are astrophysical objects that are one of the simpler solutions of Einstein's theory of general relativity, the mathematical theory that describes gravitation in terms of curved space and time (or just "spacetime," for short). The layout, numbering and themes in each numbered section of the verse reflect the different causally distinct regions of the mathematical solution of a black hole that has existed for all time (this will be described in more detail below). The emphasis here "on all time" is to distinguish it from one that formed from the collapse of a massive star, like many of the astrophysical black holes in our universe, which have a different causal structure from black holes that have always existed.

Karl Schwarzschild and the Gravitational Field of a Star

The first player in this story (and the first name in the title) is that of Karl Schwarzschild (1873–1916). Schwarzschild was an astronomer and physicist who was born and spent his short academic career in Germany. He had broad research interests in both astronomy and physics, though here the focus will be on his contributions to general relativity. In 1914, one year before Einstein published his completed theory of general relativity [1], Schwarzschild volunteered to serve in the German army during World War I. Despite serving in the army and suffering from an autoimmune disease, Schwarzschild was able to publish two pioneering works on general relativity [2, 3].

In 1916, he solved Einstein's field equations of general relativity exactly, and obtained the gravitational field outside of an isolated massive astrophysical object, such as a star, with the assumptions that the object is spherically symmetric (appears the same when viewed from any angle around the source) and static (it is unchanging in time). In the first paper [2], Schwarzschild did not focus on the interior of the solution, namely the matter in the star that gives rise to the gravitational field he had calculated. This inner solution (namely, the interior of the star) was the subject of a second paper [3], also published in 1916. Sadly, Schwarzschild's health rapidly declined in 1916; he left the Russian front where he had been fighting to return to Germany, and died only a few months after his return.

The exterior solution found by Schwarzschild had the property that as the radial coordinate used by Schwarzschild approached zero, the area of the spheres of the coordinate radius approached a constant, nonzero value. Moreover, the measure of distance in the radial direction (a component of the metric) diverged in this limit. While this might seem troubling, the physical distance where this would occur, for a typical star, would be much smaller than the size of star; thus, Schwarzschild's inner solution describing the matter and gravity within the star would instead be the valid description of the star, and it does not have any divergences.

A Collapsing Star and Einstein's General Relativity

Many years later, however, the possibility of very dense, compact stars began to be studied in more depth. This included a 1939 paper by Oppenheimer and Snyder [4] on the stars that would collapse to have a surface area less than that of the spheres where Schwarzschild's exterior solution became poorly behaved. Though it was not fully addressed in the paper in Reference [4], the result highlighted the importance of understanding the strange behavior of Schwarzschild's exterior solution at this critical radius, which has been described as the "Schwarzschild singularity" though is now called the "Schwarzschild radius."

A poor choice of coordinates can obscure the underlying physical properties of the gravitational fields. An analogous example occurs when ascribing coordinates to points on the surface of the Earth. The North and South Poles have the property that the longitude becomes ill-defined at these points, while there is nothing intrinsically singular about the poles (a rotated set of coordinates could have these singular points appear on the equator, for example).

Einstein's general relativity has the property that the underlying equations of the theory can be expressed in a wide range of possible coordinates, and the structure of the equations has the same underlying mathematical form in all these coordinates. Specifically, Einstein's equations dictate that space and time are curved in response to the distribution of matter and energy within, and the curving of space-time determines the properties of the gravitational field, in the sense that light or particles with negligible masses follow the shortest paths in this curved spacetime. Thus, the divergences at the Schwarzschild radius could be the result of a poor choice of coordinates rather than a singular feature in the underlying spacetime at that particular radius. Work many years later would show that this was indeed the case.

Eddington, Finkelstein and the Event Horizon

A key step in understanding the exterior solution after collapse came from work by David Finkelstein (1929–2016) [5]. He found a choice of coordinates in which the singularity in Schwarzschild coordinates was removed, and the region inside the Schwarzschild radius could be studied. As a brief aside, the astronomer Arthur Eddington (1882–1944) had earlier computed the transformation of the time variable used by Finkelstein, though in a different context [6]. These coordinates, thus, are called Eddington-Finkelstein coordinates.

Finkelstein found, in fact, that the surface behaved like a one-way membrane, in the sense that once any particle or light passed through this surface, it would inevitably reach the origin (zero distance from the center). This surface was subsequently called the "event horizon" and the full solution, both interior and exterior was called a "black hole." Moreover, a light ray directed outward from the event horizon would be frozen in place, never escaping nor falling into the black hole. At the center of the interior of the horizon is a true singularity (meaning space is infinitely curved). Everything that falls into the event horizon would reach the singularity and be destroyed by the large tidal forces near it. The singularity is a prediction of classical general relativity, but it is likely that quantum mechanical effects would change its structure (though analyzing the singularity would require a theory of quantum gravity, and there is no consensus, at the time of writing, that there is a theory of quantum gravity that describes nature).

The remarkable conclusion of Finkelstein's paper was that particles can freely pass from the exterior of the Schwarzschild radius to the region inside the event horizon, and that the region within the horizon cannot communicate with the exterior. Thus, there are two distinct regions of the spacetime: the region from the Schwarzschild radius to infinite distances (referred to as region I), and the region from the Schwarzschild radius to the origin (denoted by region II). The experience of observers in regions I and II is described in the verse above in the stanzas that are labelled by those numbers, respectively. One strange feature of Schwarzschild's coordinates inside the event horizon is that the radial coordinate plays the role of a time-like variable whereas the time variable behaves more like a distance, in a mathematical sense (for both radial and time coordinates). Namely, although the radius still determines the surface area of spheres in the spherically symmetric Schwarzschild solution, surfaces of constant radius behave like surfaces of simultaneity rather than surfaces of constant distance. The surfaces of constant Schwarzschild time coordinate behave like surfaces of constant distance inside the event horizon; however, the surfaces of constant Eddington-Finkelstein time do behave like surfaces of simultaneity, and they would give a more typical notion of the time as experienced by a family of displaced observers who fall into the black hole.

Kruskal, Szekeres and the "Eternal" Black Hole

The interior of the event horizon and the exterior turn out not to be the entirety of a black hole solution for a black hole that has existed for all time (often called an "eternal" black hole). In fact, there are two other regions of the spacetime which have other interesting properties. These new regions were found through a transformation to new coordinates that covered a larger patch of the curved space than what Finkelstein had computed.

Martin Kruskal (1925–2006), a mathematician and physicist, and separately George Szekeres (1911–2005), a mathematician, independently found a coordinate transformation that revealed the other two regions of an eternal black hole [7, 8]. First, there is the region that behaves like a time-reversed version of a black hole. Namely, it is a region within the Schwarzschild radius, but any particle or pulse of light within this region must be ejected from the region in a finite time. This is region III described in the verse, which gets called a "white hole," because nothing can remain in it (and would thus appear to be bright, if there were such an object in our universe).

A second curious feature of the Kruskal-Szekeres extended spacetime is that anything that is forced out of the white hole can end up in Schwarzschild's exterior solution (region I), or a second independent exterior region, most commonly denoted region IV. The second exterior region is disconnected from the first; no information can be communicated between the two. However, these two regions share a common black hole interior region within the Schwarzschild radius. Although the two regions cannot directly communicate, observers that plunge into the black hole, could, in principle, briefly meet again before they are crushed at the singularity (though they would be unable to coordinate such a meeting). Thus, there are four regions of spacetime with their own causal structure, which is why the verse has four stanzas.

The orientation of the different regions with respect to each other is typically shown using a spacetime diagram in Figure 1. In this diagram, the Kruskal-Szekeres time coordinate (measured using the light travel distance, rather than seconds, for example) is depicted as the vertical direction, so that the past of a point in space is located below it and the future is located above it. The Kruskal radial distance runs out from the central point in the figure from right to left. This figure represents just a single set of angles around the spherically symmetric source (analogous to a fixed point of longitude and latitude on the surface of the Earth). Given the time units used here, light rays that propagate inward or outward (at the fixed angle depicted in the diagram) are positively or negatively sloped 45-degree lines. The dashed black lines show two very special such light rays: they form the boundaries between the four regions I–IV, and they mark the horizons of the white hole in the past of the central point and the black hole in the future of this point. The solid black curved lines represent the past singularity of the white hole (from which everything in region III moves away) and the future singularity of the black hole (which everything in region II approaches). The regions below and above these black curves, respectively, are shaded gray because nothing in classical general relativity can pass through the singularity. To mirror the structure of the spacetime diagram, the different stanzas were arranged in the diamond shape with the numbering not sequential, but chosen to match the numbering of the different regions of the diagram.

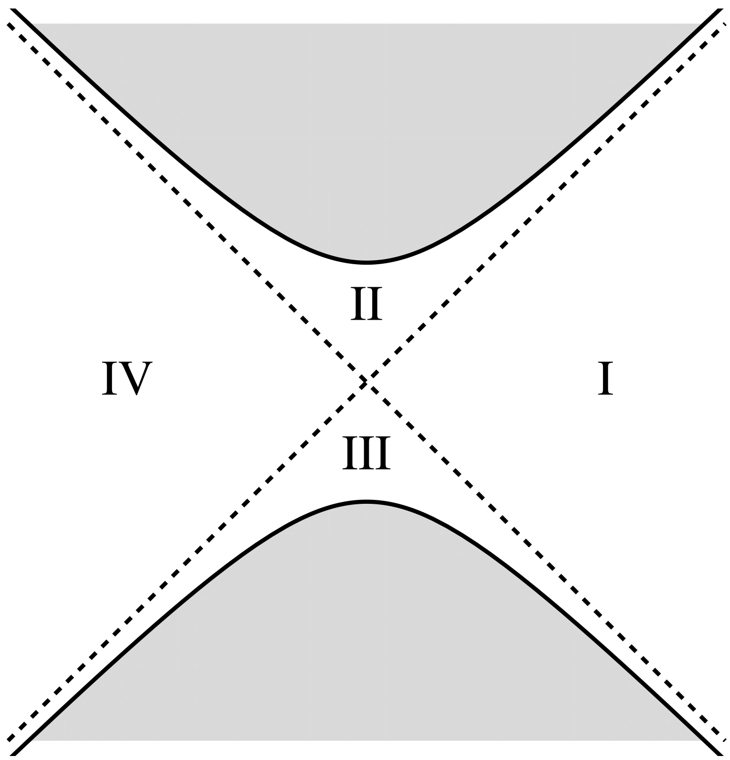

Figure 1: A spacetime diagram, with the Kruskal time (measured in light-travel distance) being the vertical direction and the Kruskal radial coordinate being the horizontal direction. Light rays follow positively or negatively sloped 45-degree lines in this diagram as a result of this coordinate choice. Region I is the exterior of the Schwarzschild radius, and is the universe that is covered by Schwarzschild's exterior solution. The region II is the interior of the black hole's (future) event horizon (the upper black dashed lines), which contains the singularity at zero radius (the upper solid black curve). Anything that enters region II will encounter the singularity, where it will be destroyed in classical general relativity. The gray region above the singularity is not part of the spacetime (and similarly, below in region III). The Eddington-Finkelstein coordinates cover both regions I and II. Region III, which mirrors region II, is the white hole, where everything above the past singularity (the lower solid black curve) will escape into region I or IV. The final quadrant, region IV is a copy of region I, which cannot communicate with region I, though like region I, it has a white hole horizon towards the past and the black hole horizon towards the future.

Conclusions

The verse and the associated discussion described some of the properties and the history of an eternal, non-rotating black hole. Of the different black-hole solutions, the non-rotating one is the simplest. A black hole that has existed for all time permits the additional white-hole and second exterior solution, which are unusual properties of the solution. There is compelling evidence for the existence of black holes in our universe, but these black holes have not always existed; rather, they formed from the collapse of stars, mergers of other black holes, or conceivably from the gravitational collapse of over-densities early in the universe. Such black holes would have a distribution of matter in their past (rather than a white hole) and there is no reason to believe that there would be a second exterior region either. After forming from gravitational collaps, they would look more like the regions I and II that are covered by Eddington-Finkelstein coordinates. It is thought that rotating black holes that form astrophysically would also have a similar causal structure.

For those interested in learning more about black holes, there are many textbooks that one can consult. Three that I have used are the following: [9–11]. A less technical introduction can be found in [12].

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